

A Pyramid Of (Formal) Software Verification

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I'm Going To Say Some Things...



...but not as many things as I would like to say

- These slides:
<http://polgreen.github.io/pdfs/pyramid-slides.pdf>
- Tutorial paper:
<http://polgreen.github.io/pdfs/pyramid-paper.pdf>
- Tutorial on youtube:
<https://youtu.be/B1GZuQIESRU?si=qEzNGt6wvqtq91m1>

- 1 Verification
- 2 Automatic Verification
- 3 Under-Approximate
- 4 Human-Assisted
- 5 Over-Approximate
- 6 Bringing It All Together

What is Verification?

A **process** that produces **evidence** that
a **system** complies with a **specification**.

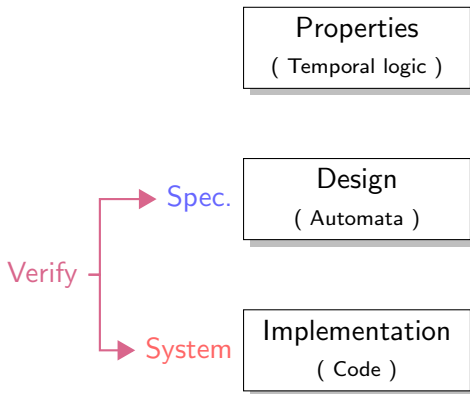
The Role of System and Specification

Properties
(Temporal logic)

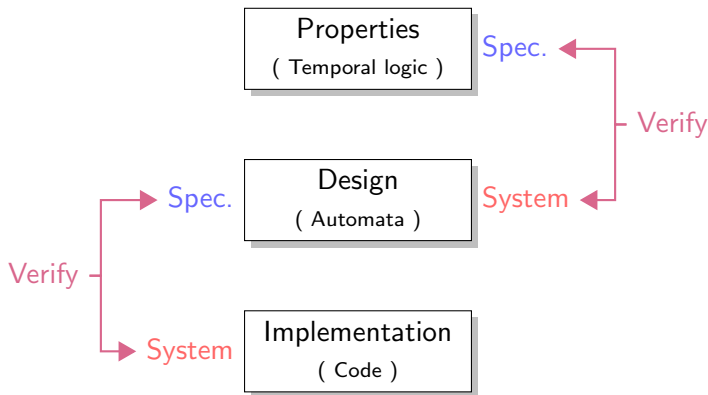
Design
(Automata)

Implementation
(Code)

The Role of System and Specification



The Role of System and Specification



Verification vs. Validation

Verification

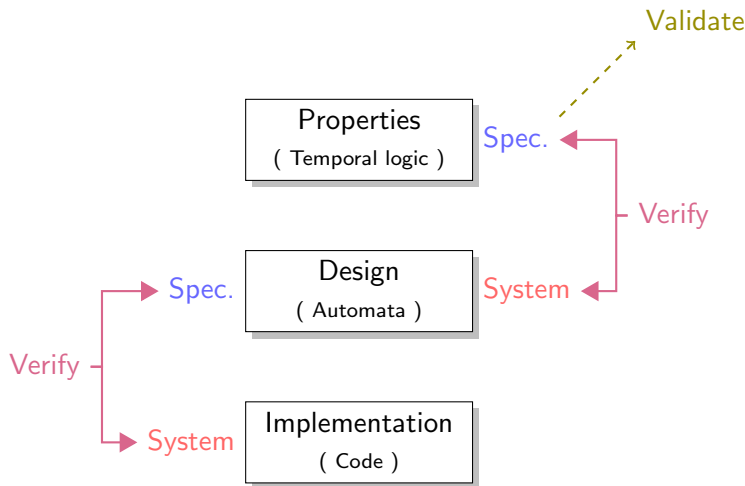
Are We Building The Thing Right?

Validation

Are We Building The Right Thing?

“Correctly building the wrong thing!” – verification without validation!

The Role of System and Specification



When To Verify

Writing
Verified
Software

VS.

Verifying
Written
Software

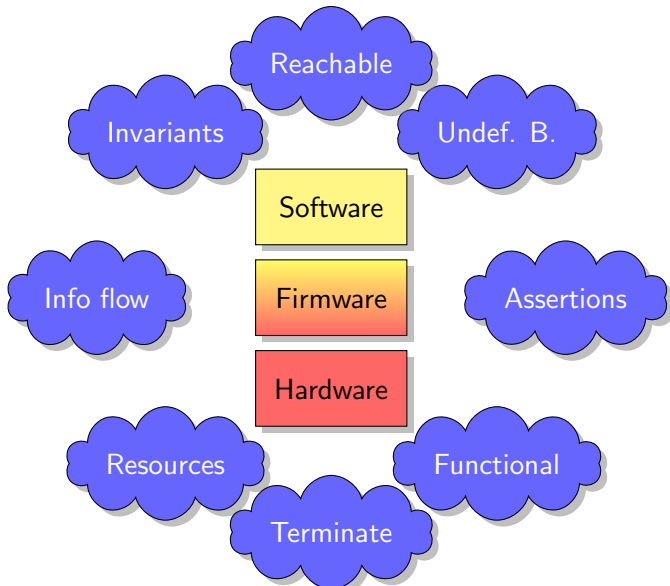
System and Specification

Software

Firmware

Hardware

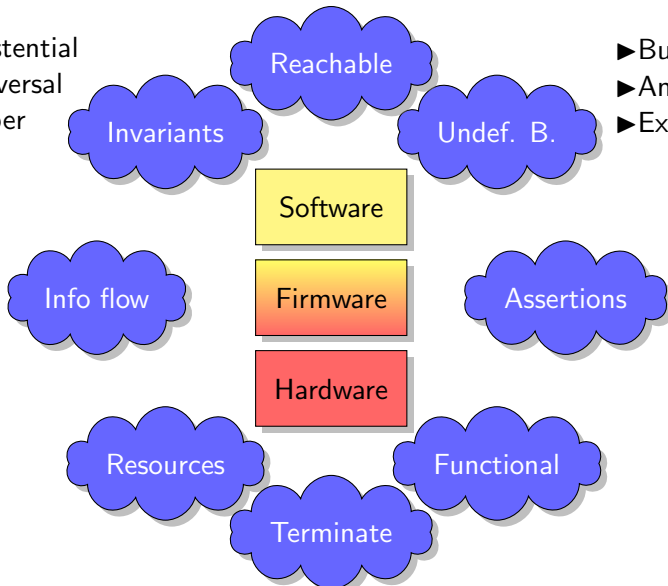
System and Specification



System and Specification

- Existential
- Universal
- Hyper

- Built in
- Annotation
- External



Why bother?



Verification

oooooooo●ooooo

Automatic Verification

oooooooooooooooo

Under-Approximate

oooooo

Human-Assisted

oooooooo

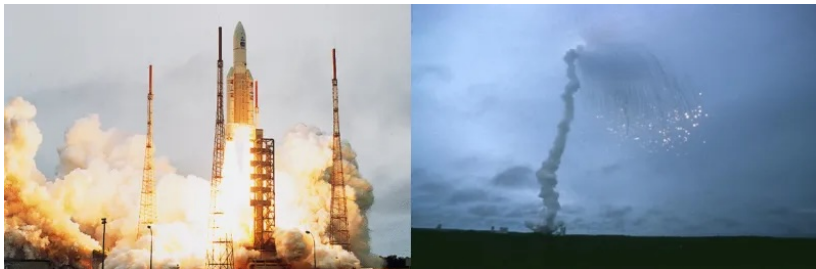
Over-Approximate

oooooo

Bringing It All Together

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ooooooo

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ooooooo

Why bother?



Why bother?



➡ FROM LEFT: MEDIA STAND IN A PARKING LOT, WAITING FOR A PRIUS TO RUN AWAY;
RECALLS KEPT TOYOTA SERVICE BAYS PLENTY BUSY

Why bother?



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oooooooooooooooo●

Automatic Verification

oooooooooooooooooooo

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Bringing It All Together

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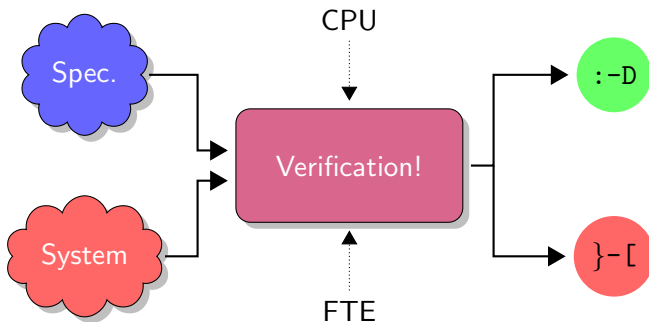
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Verification Tools

The Key Question

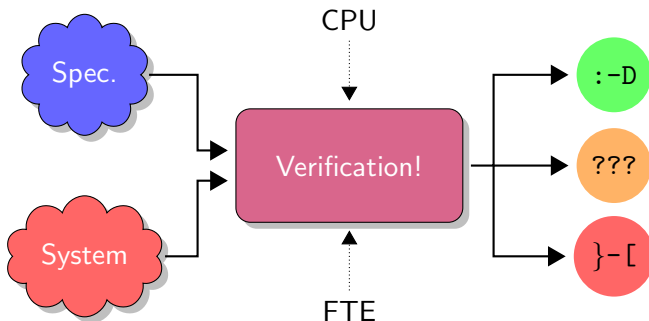
Does (every run of) system P satisfy specification S ?



Verification Tools

The Key Question

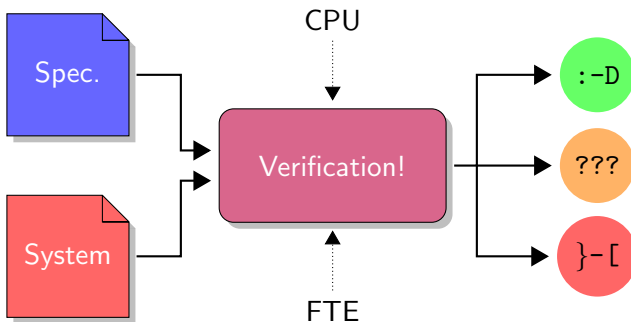
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Verification Tools

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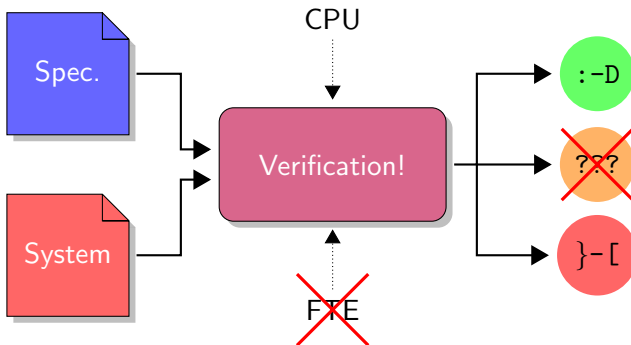


If program and specification are formal \rightarrow can automate!

The Ideal Tool

The ideal verification tool ...

- fully automated
- never misses bugs
- never gives false alarms



The Key Trade-Off

Turing's Work Implies ...

It is **impossible** to build an *automatic* verification system for:

- *any* specification that includes reachability
- *all* software including loops / recursion



The Key Trade-Off



The Key Trade-Off

Choosing Verification Tools

- 1 Pick *two* attributes based on project:
automatic, no missed bugs, no false alarms
- 2 Use CPU and human effort to get enough of the third.
(for your *specific* software/specifications.)

The Key Trade-Off

Choosing Verification Tools

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OR

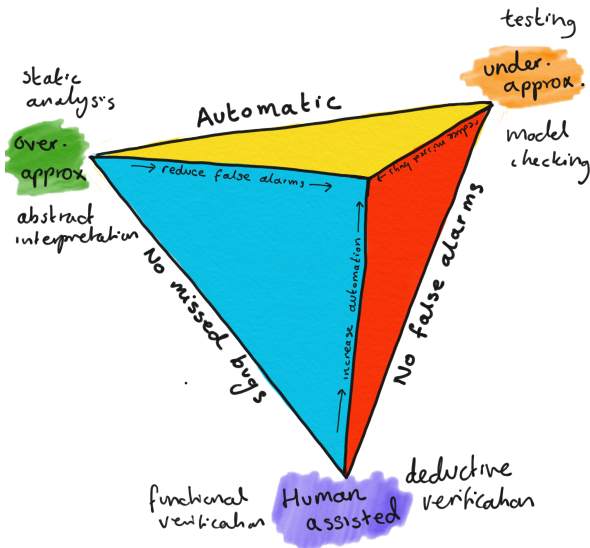
Work out which one you hate the least:

False Alarms Sentencing potential bug reports.

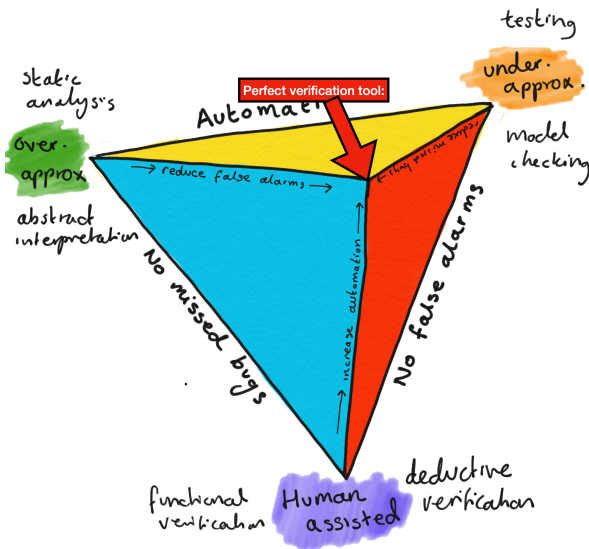
Missed Bugs Measuring coverage and writing harnesses.

Manual Supplying annotation or proof.

The Pyramid Model of Software Verification

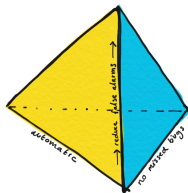


The Pyramid Model of Software Verification



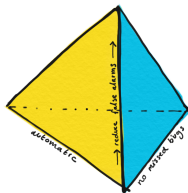
Six Schools: Over-approximate

- Static Analysis, e.g., Lint
 - Spurious warnings are fine, as long as there aren't too many
 - Lexical scanners: look for patterns in code that are likely to be bugs



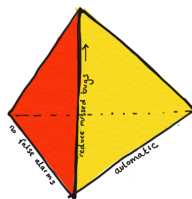
Six Schools: Over-approximate

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- Abstract Interpretation, e.g., Infer
 - Run the program with representations of sets of possible values (the domain)
 - e.g., intervals



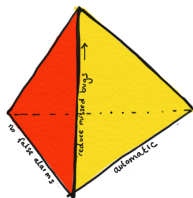
Six Schools: Under-approximate

- Testing and Symbolic Execution e.g., KLEE
 - Run the program!
 - Or, run the program with symbols instead of concrete inputs



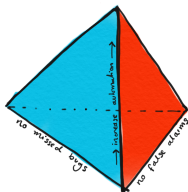
Six Schools: Under-approximate

- Testing and Symbolic Execution e.g., KLEE
 - Run the program!
 - Or, run the program with symbols instead of concrete inputs
- Model checking e.g., CBMC
 - Build a model of the program.
 - Use the model to build a formula that represents all paths in the program.
 - Use a SAT solver (or BDDs) to see if there is a path that violates the spec.



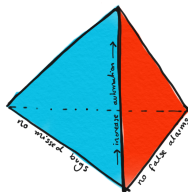
Six Schools: Human-assisted

- Deductive Verification e.g., SPARK
 - Describe the set of states with a predicate
 - Use logic to link these together (e.g., Hoare logic)
 - Use proof by induction for loops (unbounded proof!)



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- Functional Verification e.g., Agda
 - Define a programming language that *only* lets you build correct programs
 - Specifications are captured in types
 - Only good for verifying programs as you write them

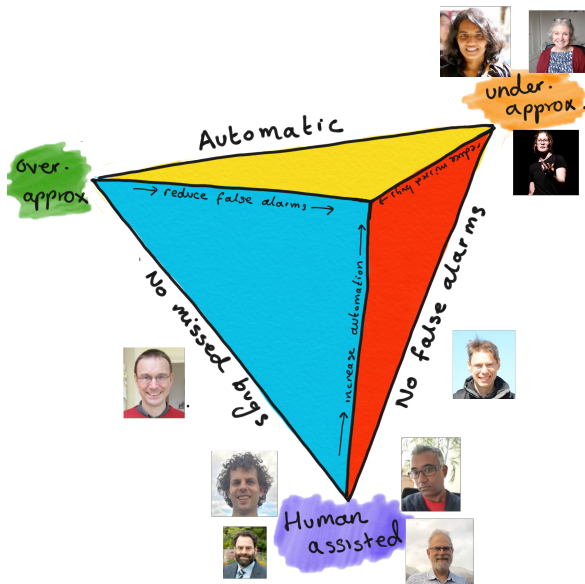


Six Schools

Program	Over-Approximate		Under-Approximate		Human-Assisted	
	Static Analysis	Abstract Interpretation	Testing & Symbolic Execution	Model Checking	Deductive Verification	Functional Verification
Program	Procedural or O.O.	Procedural	Procedural or O.O.	Procedural or O.O.	Subsets of procedural	Functional
Common Means of Specification	Builtin	Annotation linked to the abstraction	Generally annotation	Annotation or external	Annotation	Type as annotation
Common Type of Specification	Data flow, aliasing, type, shape, taint	Value, shape, resource, data flow	Value, WCET, resource	Value, temporal, modal, liveness	Value, shape, termination, resource	Type, termination
Mathematical Foundations	Ad-hoc / operational semantics	Order theory	Ad-hoc / transition systems	Transition systems	Logic	Type theory
User Skill Required	Minimal	Low/Medium	Low	Medium	High	Very high
Compute Required	Minimal	Low/Medium upwards	Medium upwards	Medium/High upwards	Low/Medium	Low
Typical Output	Algorithm dependent	Alarms or abstract domains	Error traces	Error traces	Proof or local counter-examples	Type-checking errors
Major Systems	Lint[?], Coverity[?], Fortify[?], FindBugs[?], CPPLCheck[?]	Astrée[?], Polyspace[?], Infer[?], Code Contracts[?]	CREST[?], JPF[?], Pex[?], KLEE[?]	CBMC[?], Blast[?], *SMV[?], CPAchecker[?]	SPARK[?], Dafny[?], Frama-C[?], Malpas[?], Esc/Java[?]	Coq[?], PVS [?], Agda[?], Isabelle/Hol[?]

Table: Cultural attributes of the six schools.

Six Schools



Running Example: Formalise and Verify

Spec.

- 1 All array accesses in bounds
- 2 Returned last is in a
- 3 If found then $a[\text{last}]$ is target

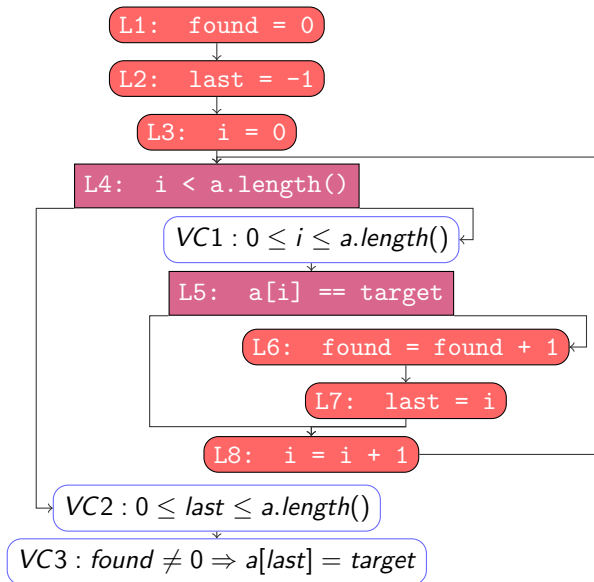
System

```
(int, int)
count (Array a, int target)
{
    int found = 0;
    int last = -1;
    int i = 0;

    while (i < a.length()) {
        if (a[i] == target) {
            found = found + 1;
            last = i;
        }
        i = i + 1;
    }

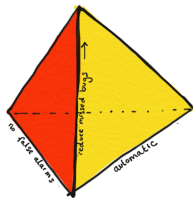
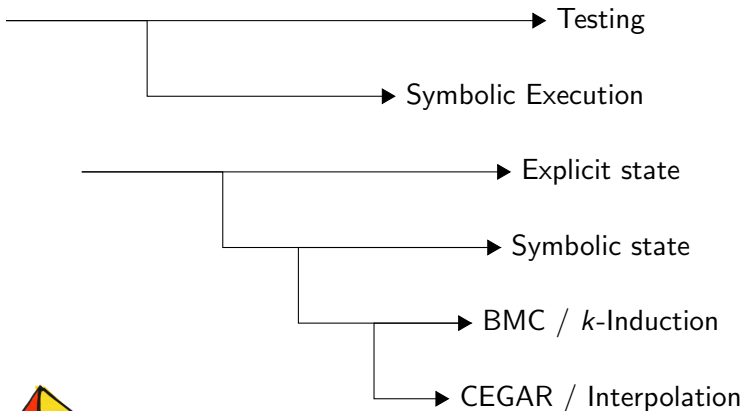
    return (found, last);
}
```

Running Example: Formalise and Verify

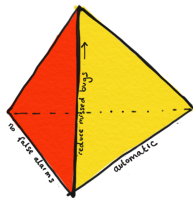
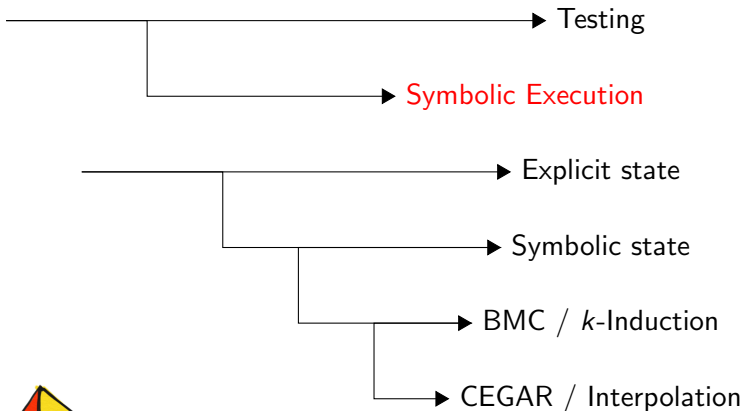


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Under-Approximate Family Tree



Under-Approximate Family Tree



Symbolic Execution: Foundations (Ideas)

Can run one test case which gives an execution trace...

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Can we generalise this to “similar” traces?

Symbolic Execution: Foundations (Ideas)

Can run one test case which gives an execution trace...

Can we generalise this to “similar” traces?

Use logic to describe a *set* of traces (that take the same path).

Symbolic Execution: Foundations (Symbols)

X is set of variables, $I \subset X$ is a set of input variables

$Expr(I)$ is the set of expressions over I

$Prd(I)$ is the set of predicates over I

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then update $Env(v)$ with $Env(x) \text{ op } Env(y)$.

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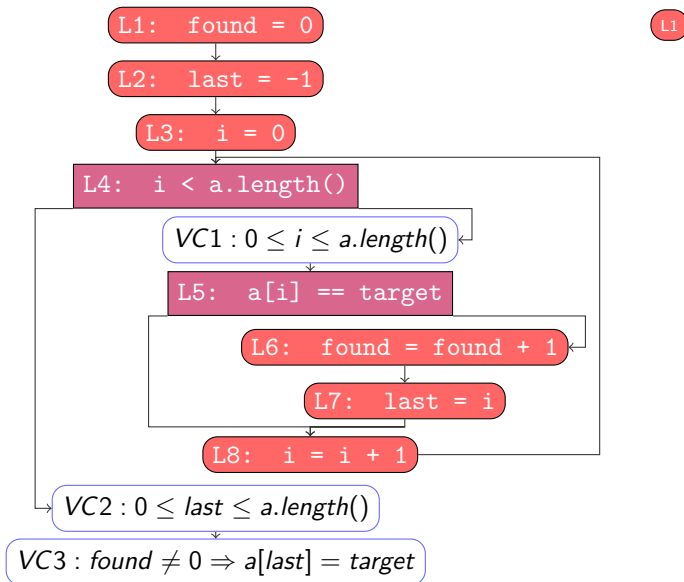
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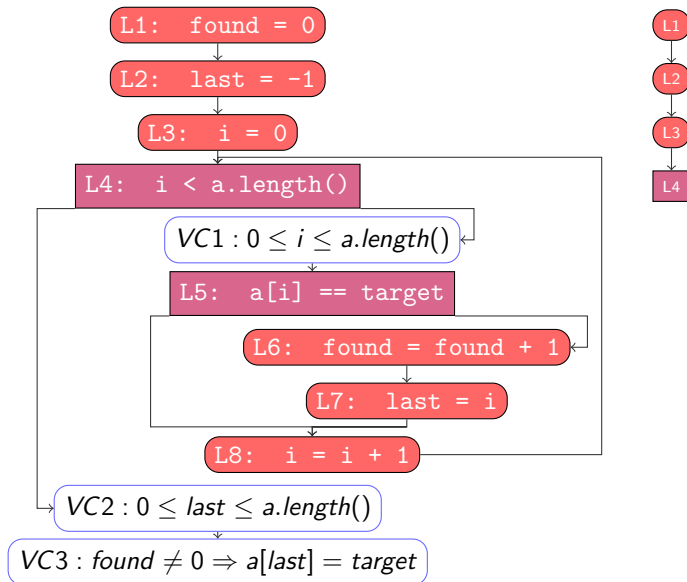
Branch If branching on $x \text{ rel } y$
 then add $Env(x) \text{ rel } Env(y)$ to PC .

Check Satisfiability check PC .
 If *unsat* then discard the trace.

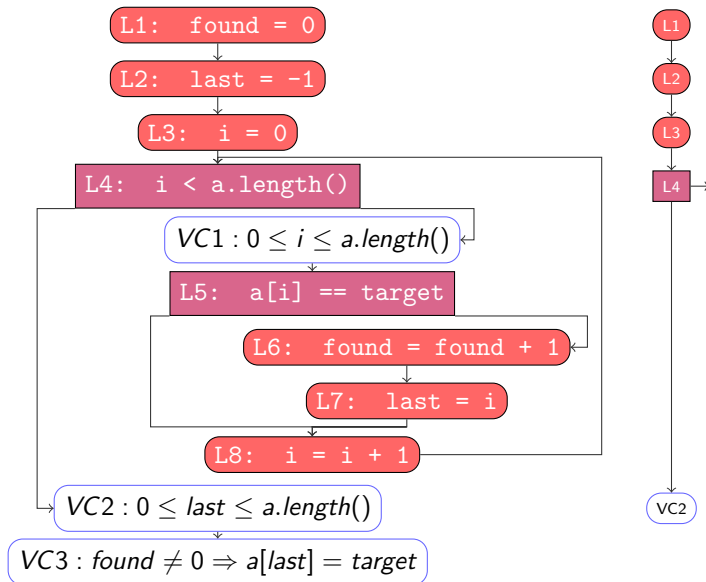
Symbolic Execution: Running Example



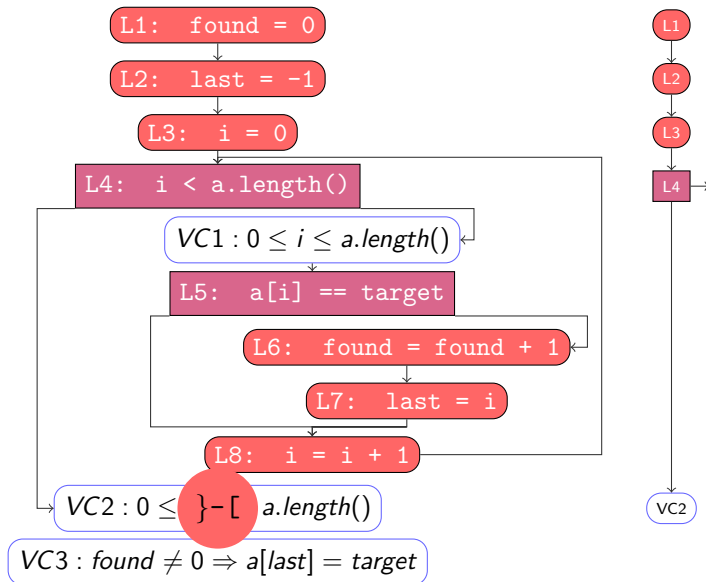
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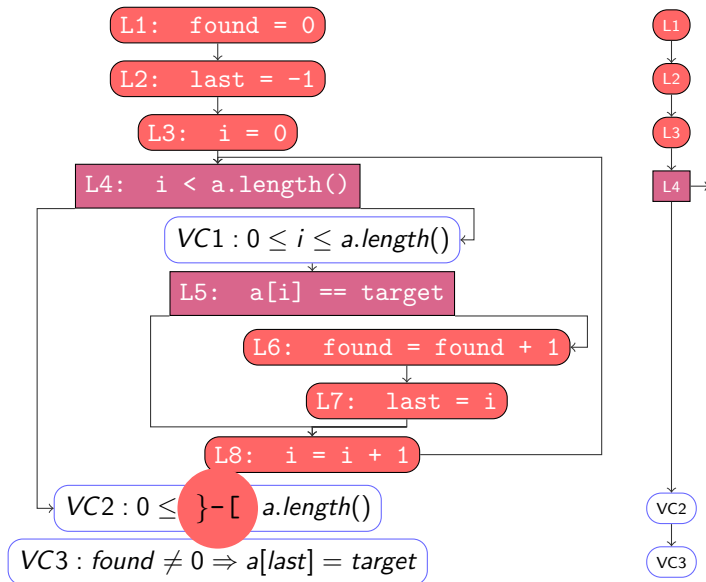
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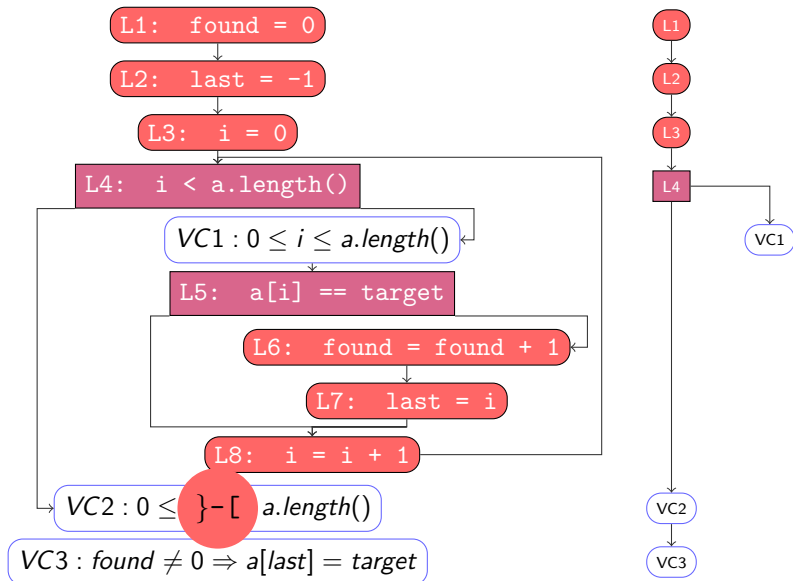
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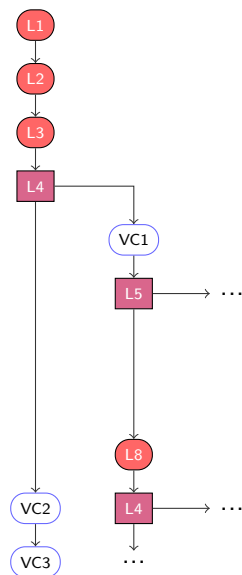
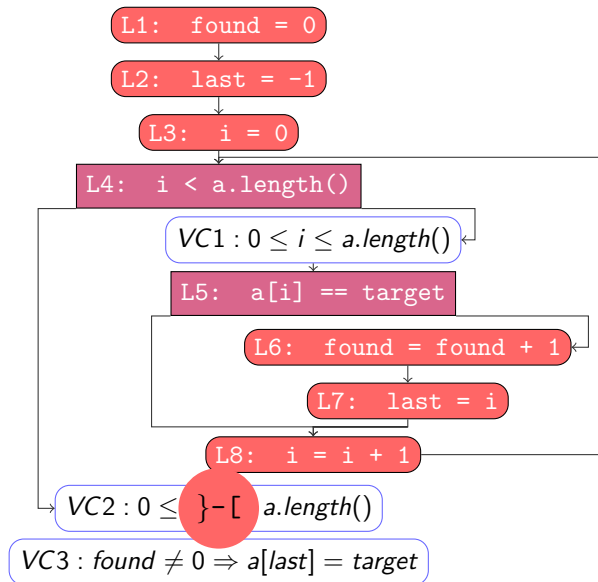
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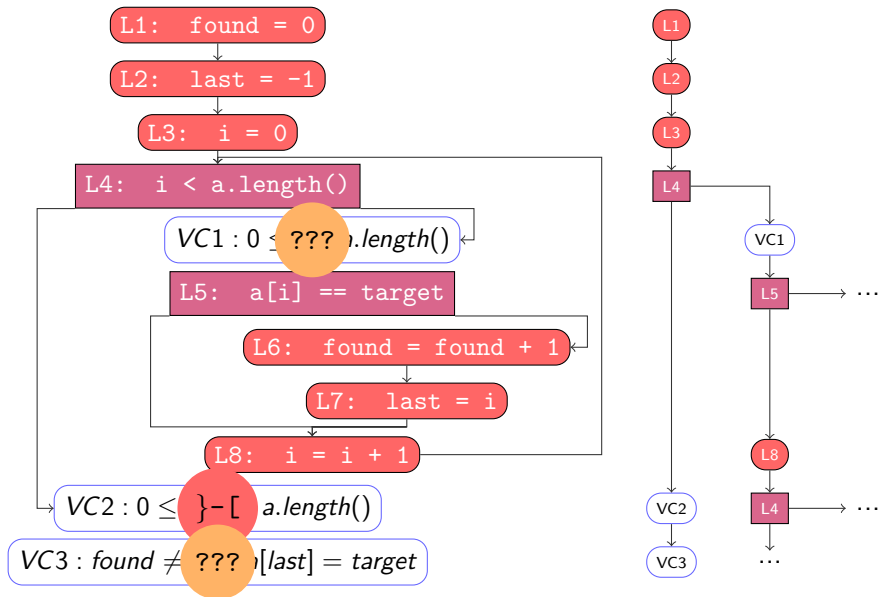
Symbolic Execution: Running Example



Symbolic Execution: Running Example



Symbolic Execution: Running Example



Symbolic Execution: Pros and Cons

Pros

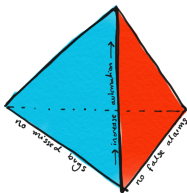
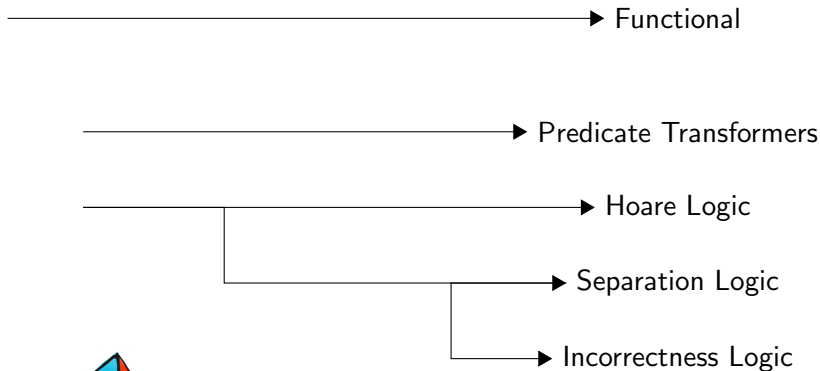
- Counter-examples!
- Concretisation
- Anytime

Cons

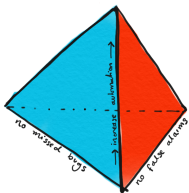
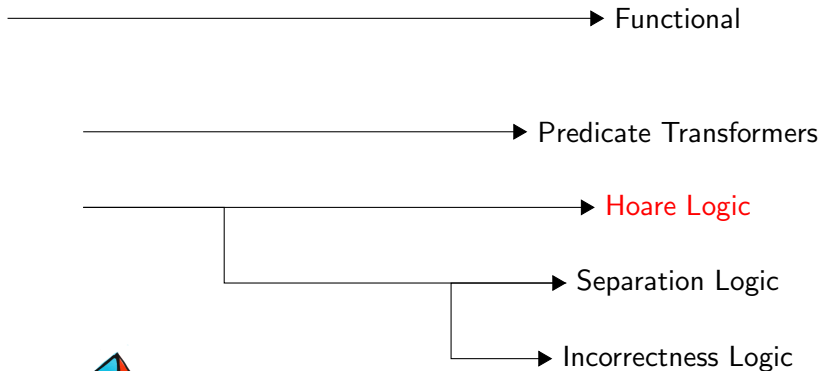
- Combinatorial explosion!
- Non-modular
- Need complete program

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Human-Assisted Family Tree



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Deductive Verification: Foundations (Ideas)

Describe the set of possible states (at a program location) with a *predicate*

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Describe the set of possible states (at a program location) with a *predicate*

Use logic to link these together...

and use **proof by induction** for loops!

Deductive Verification: Foundations (Symbols)

Hoare Triples

$$\{Pre\} \text{ Program } \{Post\}$$

“If the state of the program meets the precondition (Pre is true) then after Program has been run the state will meet the postcondition ($Post$ is true)”

Deductive Verification: Foundations (Symbols)

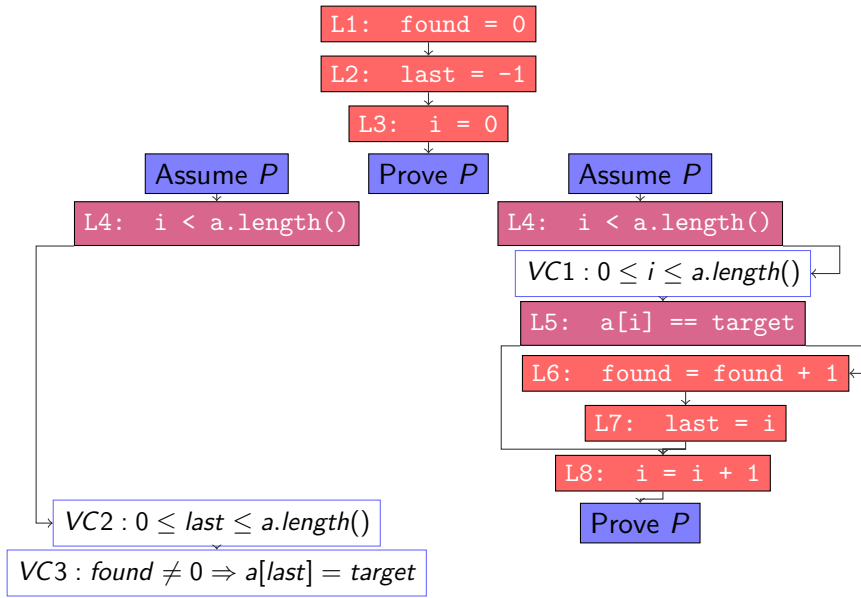
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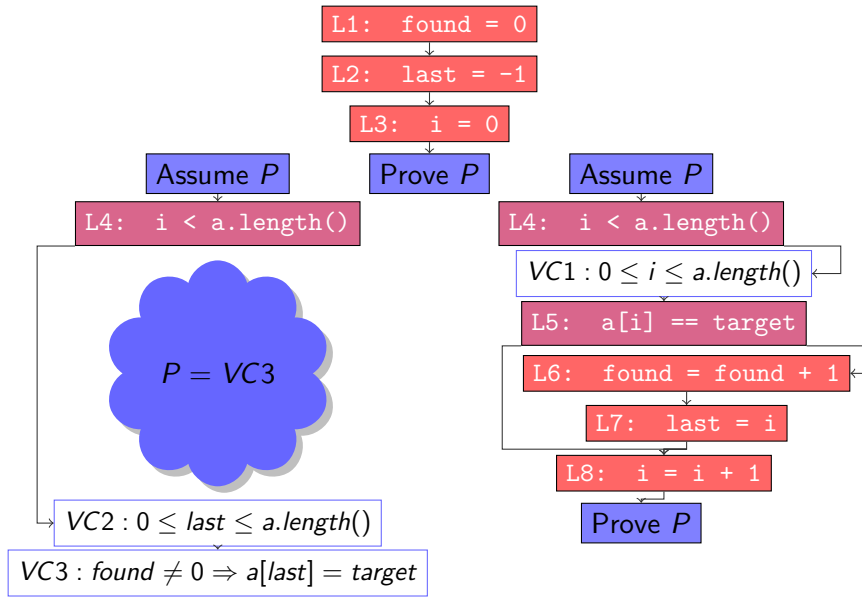
“If the state of the program meets the precondition (Pre is true) then after Program has been run the state will meet the postcondition ($Post$ is true)”

$$\frac{\{Inv \wedge Cond\} \text{ Body } \{Inv\}}{\{Inv\} \text{ while } (Cond) \text{ Body } \{Inv \wedge \neg Cond\}}$$

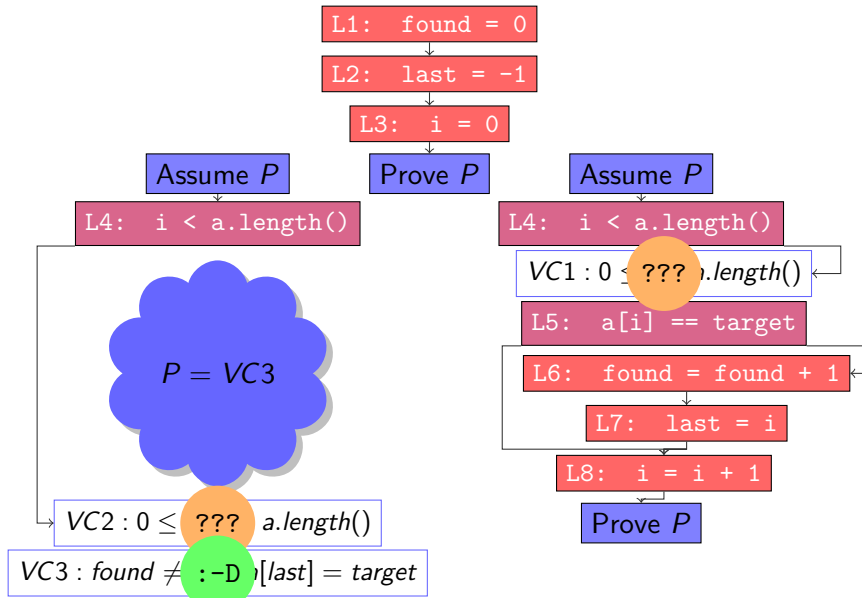
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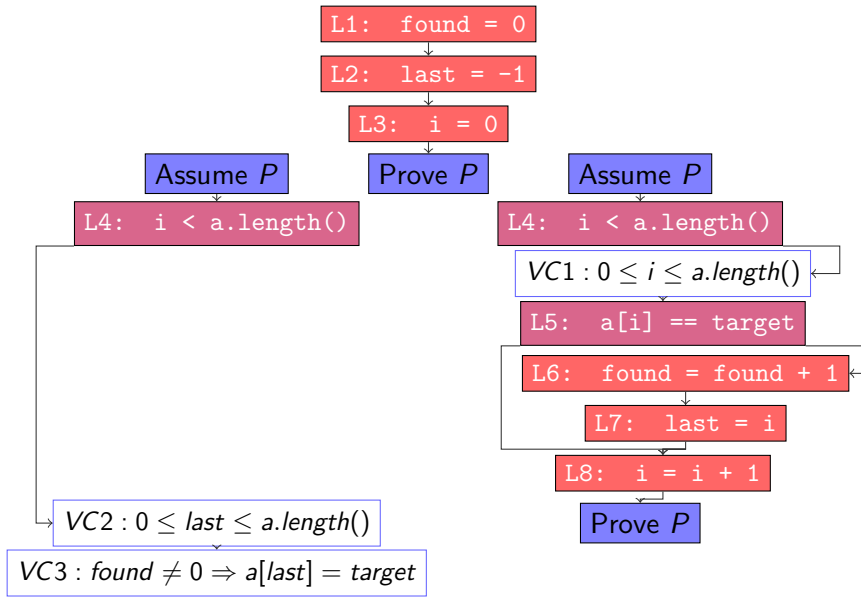
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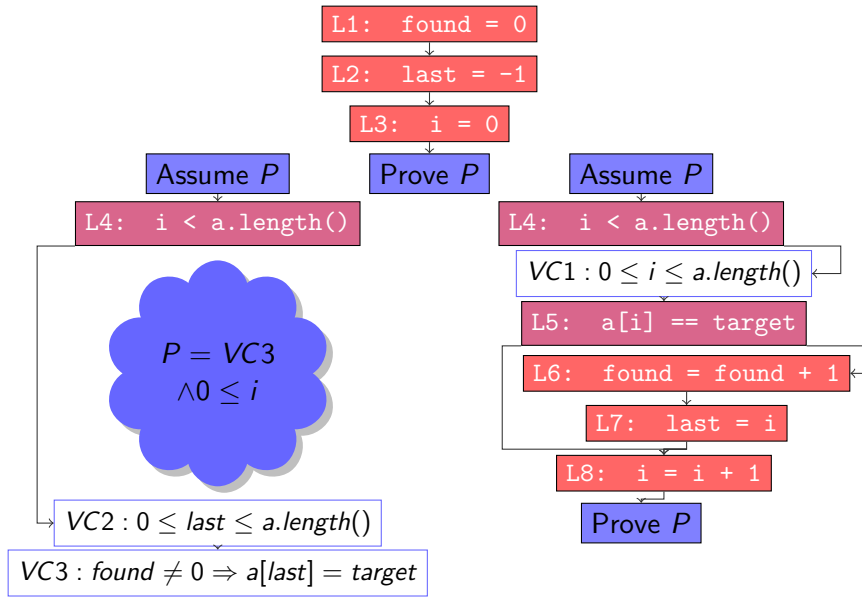
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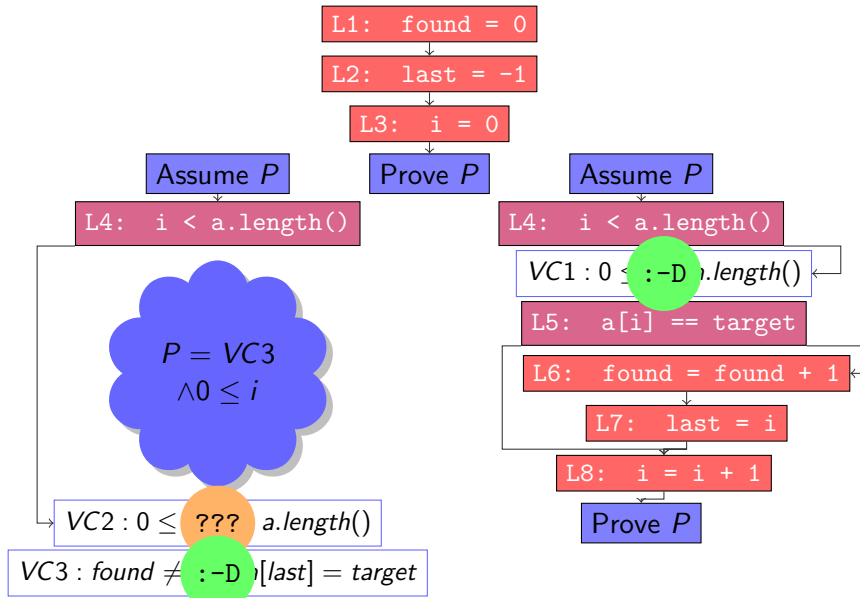
Deductive Verification: Running Example



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Deductive Verification: Running Example



Deductive Verification: Pros and Cons

Pros

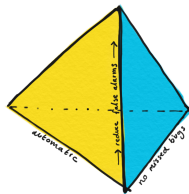
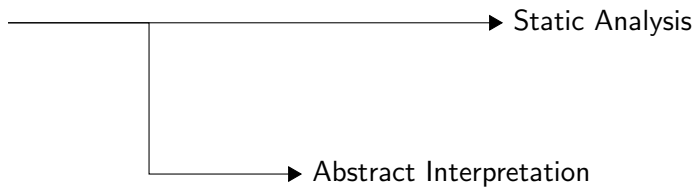
- Certainty
- Scalable (compute)
- Incremental

Cons

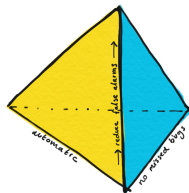
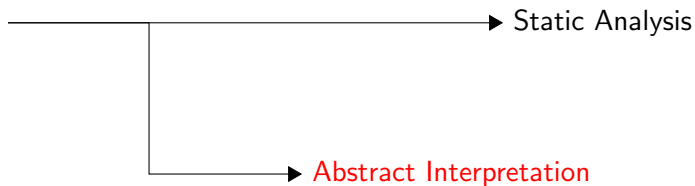
- Maintenance
- Scalable (human)
- False vs. not provable

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Abstract Interpretation: Foundations (Ideas)

Want to reason about *all* possible executions...

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What if we run the program with sets of possible values?

Abstract Interpretation: Foundations (Ideas)

Want to reason about *all* possible executions...

What if we run the program with sets of possible values?

That's too big and too slow but what if we run the program with
representations of sets of possible values?

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X is set of variables

$Env = X \rightarrow \mathbb{Q}$ is set of program states

$Instr = Env \rightarrow Env$ is set of program instructions

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$$\ll i \in [0, 4], j \in [0, 4], n \in [5, 5], m \in [5, 5] \gg \in L$$

Abstract Interpretation: Foundations (Symbols)

X is set of variables

$Env = X \rightarrow \mathbb{Q}$ is set of program states

$Instr = Env \rightarrow Env$ is set of program instructions

Representation Set L of representations $\gamma : L \rightarrow 2^{Env}$

$$l_1 \sqsubseteq l_2 \Leftrightarrow \gamma(l_1) \subseteq \gamma(l_2)$$

$$\ll i \in [0, 4], j \in [0, 4], n \in [5, 5], m \in [5, 5] \gg \in L$$

Transform $T : Instr \times L \rightarrow L$ with $f(\gamma(l)) \subseteq \gamma(T(f, l))$

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$$\ll \dots i \in [0, 4] \dots \gg \sqcup \ll \dots i \in [1, 5] \dots \gg =$$

$$\ll \dots i \in [0, 5] \dots \gg$$

Widen $\nabla : L \times L \rightarrow L$ with $l_1 \sqsubseteq l_1 \sqcup l_2, l_2 \sqsubseteq l_1 \sqcup l_2$

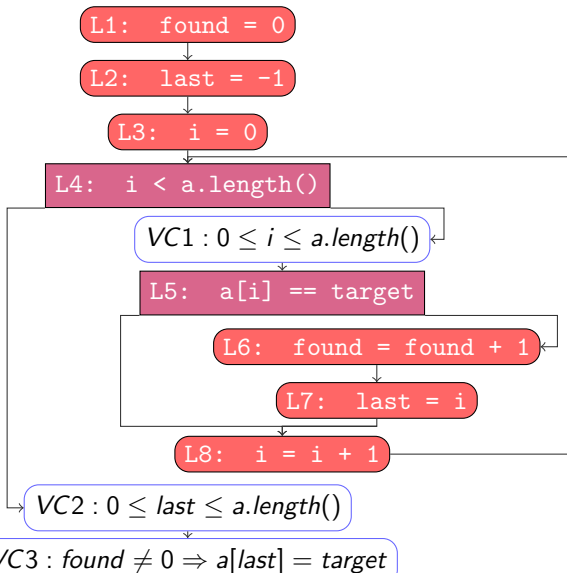
“guarantees termination”

$$\ll \dots i \in [0, 4] \dots \gg \nabla \ll \dots i \in [1, 5] \dots \gg =$$

$$\ll \dots i \in [0, \text{inf}] \dots \gg$$

Abstract Interpretation: Running Example

found last i



Abstract Interpretation: Running Example

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L1: found = 0

L2: last = -1

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L3: i = 0

L4: i < a.length()

VC1: $0 \leq i \leq a.length()$

L5: a[i] == target

L6: found = found + 1

L7: last = i

L8: i = i + 1

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VC3: $found \neq 0 \Rightarrow a[last] = target$

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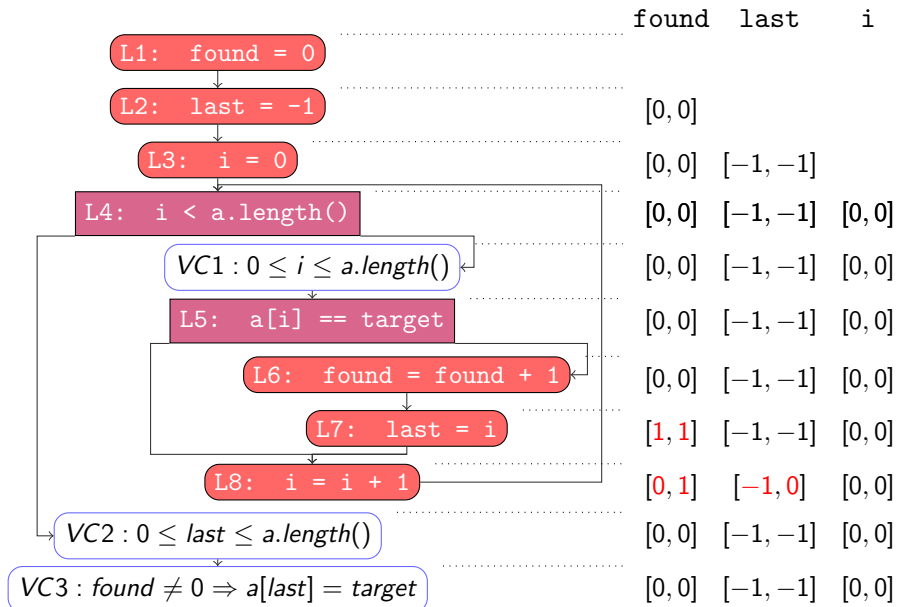
[0, 0] [-1, -1] [0, 0]

[1, 1] [-1, -1] [0, 0]

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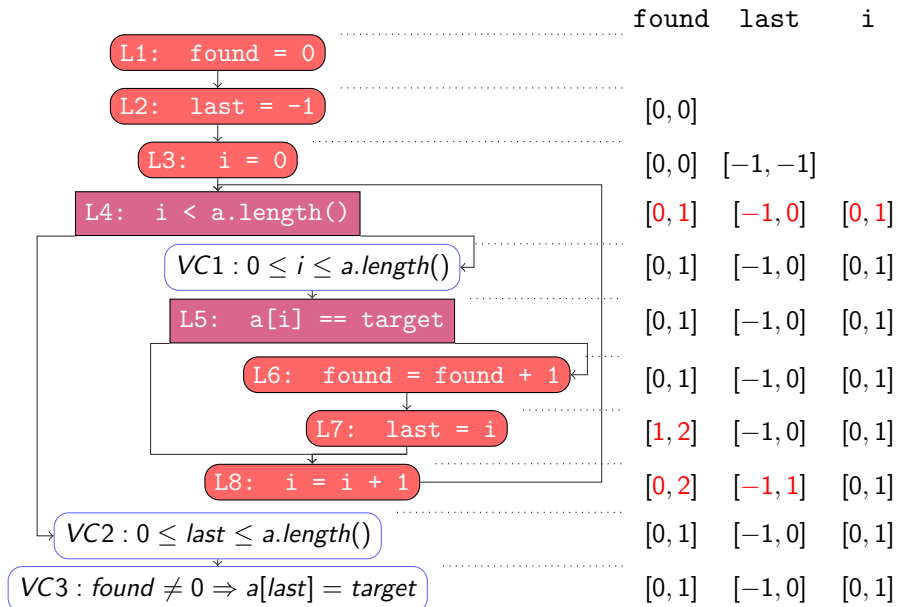
[1, 1] [-1, -1] [0, 0]

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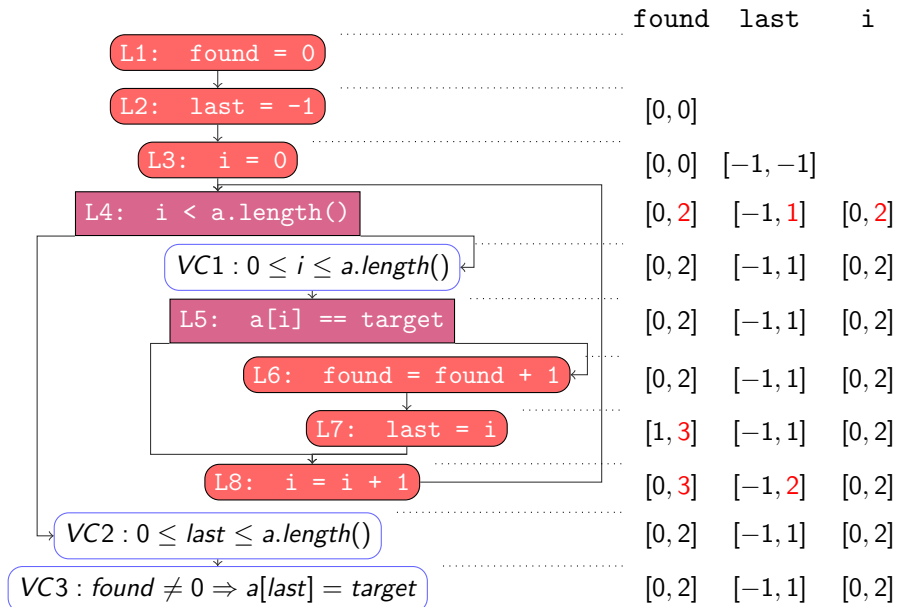
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[1, x + 1] [-1, x - 1] [0, x]

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L8: i = i + 1

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VC3: found ≠ 0 ⇒ a[last] = target

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[0, 0] [-1, -1]

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[0, x] [-1, x - 1] [0, x]

[0, x] [-1, x - 1] [0, x]

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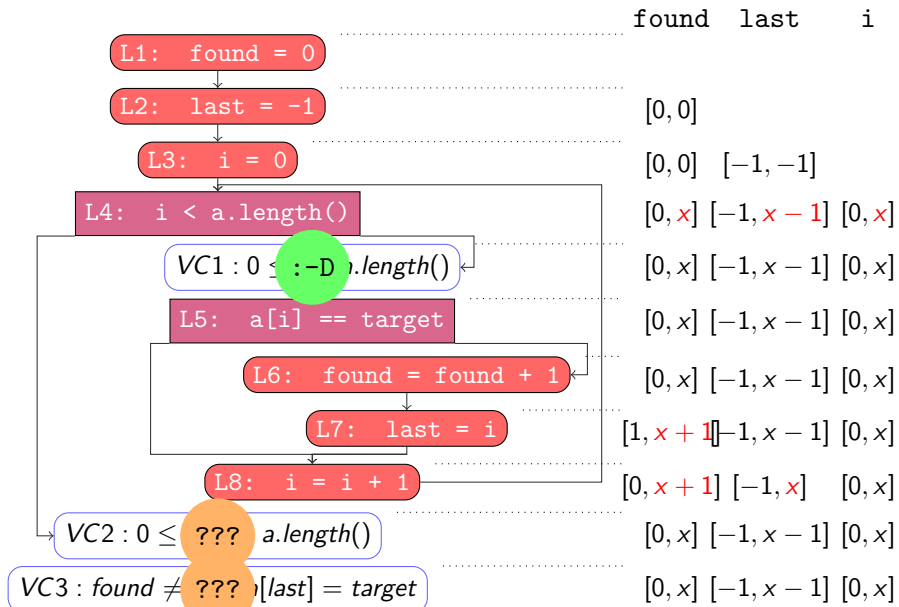
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Abstract Interpretation: Running Example



Abstract Interpretation: Pros and Cons

Pros

- Assuming independence is an overapproximation
- Can discard information about states
- Compositional / modular

Cons

- When you reach $T...$
- “Yes but why?”
- Widen is *hard*

- 1 Verification
- 2 Automatic Verification
- 3 Under-Approximate
- 4 Human-Assisted
- 5 Over-Approximate
- 6 Bringing It All Together**

Process Considerations

Key Trade-Off

No false alarms / No missed bugs / Automatic

Two for free, will a **reasonable** amount of computation and human effort give you **enough** of the third?

Considerations:

- What happens if the system doesn't meet the spec?
- Who uses the evidence? For what?
- Can the the system or spec be changed?
- Can the tools be changed?
- Are partial results useful?

Understanding Tool Evaluation

Skip To The Results and ...

Over-Approximate Number of Alarms (proxy for false alarms)

Under-Approximate Benchmarks Solved (proxy for coverage rate)

Human-Approximate LoC of Annotation (proxy for human effort)

Limits of Verification

Biased Personal Opinion

For verification to be “useful”,
the specification must be “simpler” than the system.

(What is the spec for *cosine*? What is the spec for `printf`? What is the spec for `getopt`? What is the spec for `strtod`? What is the spec for time & date?)

Conclusion : How to Pick Verification Tools

- ① What is your **system**? What is your **specification**?
Are they formal? Could they be?
- ② How finished / fixed are they?
“Writing verified software” vs. “Verifying written software”
- ③ What kinds of **evidence** support your goal?
- ④ Automatic, no missed bugs, no false alarms – **pick two!**
- ⑤ Fit the tool to the **process** or vica versa or both.

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- 3 What kinds of **evidence** support your goal?
- 4 Automatic, no missed bugs, no false alarms – **pick two!**
- 5 Fit the tool to the **process** or vica versa or both.

Thank you for your time and attention.

Resources

- These slides:
<http://polgreen.github.io/pdfs/pyramid-slides.pdf>
- Tutorial paper:
<http://polgreen.github.io/pdfs/pyramid-paper.pdf>
- Tutorial on youtube:
<https://youtu.be/B1GZuQIESRU?si=qEzNGt6wvqtq91m1>





Coverity scan: Static analysis.

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CPPCheck: A tool for static c/c++ code analysis.

<https://cppcheck.sourceforge.io/>, accessed:
2024-04-10



CREST: Concolic test generation tool for c.

<https://www.burn.im/crest/>, accessed: 2020-22-07



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