The relative powers of shared-variable and message-passing communication:

possibility and impossibility of speed-independent mutual exclusion.

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The code of A: Let s be your old account value.
Calculate $s' = s + 100$. Overwrite s with s'.

The code of B: Let t be your old account value.
Calculate $t' = t + 5$. Overwrite t with t'.

What will be your balance after these operations?

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In case starvation freedom fails, we at least seek a weaker property:

- Deadlock Freedom Once some process enters its entry protocol, some process will eventually be able to execute its critical section.

First Attempt

We can implement **await** using a busy-waiting loop.

var $turn := P$	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 **await** $turn = P$;	q_2 **await** $turn = Q$;
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Starvation Freedom? Nope! What if q_1 never finishes?

Deadlock Freedom? Nope! Same reason

Second Attempt

var *readyP*, *readyQ* := False, False	
forever do	**forever do**
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p_2 **await** *readyQ* = False;	q_2 **await** *readyP* = False;
p_3 *readyP* := True;	q_3 *readyQ* := True;
p_4 **critical section**	q_4 **critical section**
p_7 *readyP* := False	q_7 *readyQ* := False

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p_4 **critical section**	q_4 **critical section**
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Mutual exclusion is violated if they execute in lock-step (i.e. $p_1q_1p_2q_2p_3q_3$ etc.)

Third Attempt

var *readyP*, *readyQ* := False, False	
forever do	**forever do**
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Now we have a deadlock (or stuck state) if they proceed in lock step

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Peterson's Algorithm — Proof of Mutex Property

var *readyP*, *readyQ* := False, False; **var** *turn* := *P*	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 *readyP* := True	q_2 *readyQ* := True
p_3 *turn* := *Q*	q_3 *turn* := *P*
p_4 **await** $readyQ = \text{False} \lor turn = P$	q_4 **await** $readyP = \text{False} \lor turn = Q$
p_5 **critical section**	q_5 **critical section**
p_6 *readyP* := False	q_6 *readyQ* := False

Peterson's Algorithm — Proof of Starvation Freedom

var *readyP*, *readyQ* := False, False; **var** *turn* := *P*	
forever do	**forever do**
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Equivalently:
there can be arbitrary long pauses between any two instructions.

Importantly: even if one process in a race is ready to step over the finish, the other may still win.

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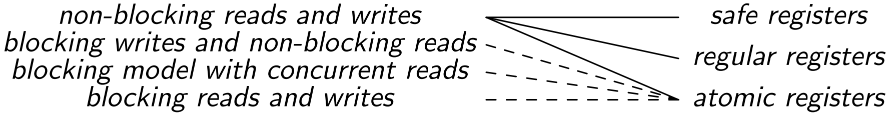

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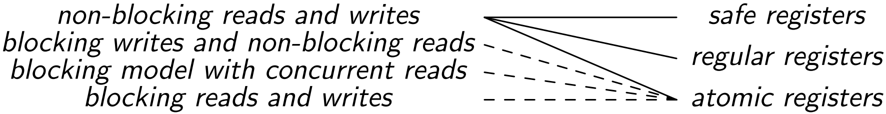

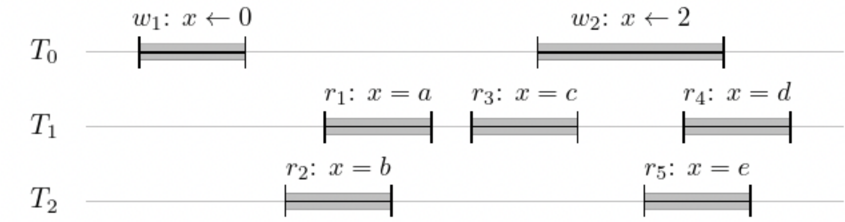

Shared variables: 4 register models

register model	*rd* block *rd*?	*rd* block *wr*?	*wr* block *rd*?	*wr* block *wr*?
blocking reads and writes	$\surd$	$\surd$	$\surd$	$\surd$
blocking model with concurrent reads	—	$\surd$	$\surd$	$\surd$
blocking writes and non-blocking reads	—	—	$\surd$	$\surd$
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blocking writes and non-blocking reads	—	—	√	√
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mutual exclusion impossible

with red registers.

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speed-independent mutual exclusion impossible

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Deadlock-free speed-independent mutual exclusion impossible with red registers.

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My claim is that atomicity is the more challenging assumption.
For then there is no speed-independent mutual exclusion protocol.

No mutex with read/write on blocking registers

$$x := v \qquad \| \qquad \textbf{await}(x = v)$$

Peterson's Algorithm

var $readyP, readyQ :=$ False, False; **var** $turn := P$	
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Peterson's algorithm without busy wait

repeat forever

$$\left\{\begin{array}{ll}
\ell_1 & \textbf{noncritical section} \\
\ell_2 & \mathit{ready}[i] := \mathit{true} \;\oplus\; \textsc{send}(\mathit{ready}[i] = \mathit{true}) \\
\ell_3 & \mathit{turn} := j \;\oplus\; \textsc{send}(\mathit{turn} = j) \\
\ell_4 & \textbf{read}(\mathit{ready}[j] = \mathit{false}) \;\parr\; \textsc{receive}(\mathit{ready}[j] = \mathit{false}) \;\parr \\
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\end{array}\right.$$

Here i is the displayed process and j the other one.

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$\mathbf{read}(\mathit{turn} = i) \;\parr\; \textsc{receive}(\mathit{turn} = i)$
ℓ_5 **critical section**
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Here i is the displayed process and j the other one.

This is a deadlock-free speed-independent mutual exclusion algorithm.

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For mutual exclusion to be possible, there must be a variable *readyP* that is written by Proc. P to request entry to CS.

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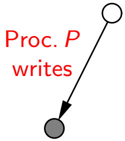

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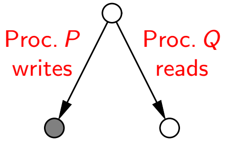

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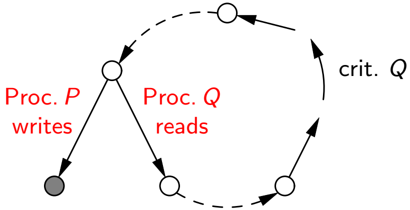

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Peterson's Algorithm — Proof of Starvation Freedom

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Peterson's algorithm — counterexample to starv. Freedom

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A classification of message passing mechanisms

asynchronous communication		$\rightarrow$		**synchronous communication**
one recipient		$<$		**multiple recipients**
interrupting receipt		vs.		**dedicated receipt**
non-blockable sending	$<$	**partly blockable**	$>$	blockable sending
choice-free	$<$	input-guarded choice	$<$	**mixed choice**
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interruptible		vs.		**uninterruptible**

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Starvation-free speed-independent mutual exclusion is possible with

- interrupting receipt
- interruptible communication
- broadcast communication with instantaneous choice

but not with any other message passing paradigm classified here.